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ABSTRACT

This study describes the emerging mathematical understanding of one student enrolled in an elementary mathematics content course. Through the medium of her journal entries, a portrait of Elena's struggle to overcome her difficulty with mathematics and construct conceptual knowledge emerges. Though these struggles are not unique, Elena's story is both poignant and extraordinarily compelling. She gives voice to the child within struggling to make sense of this "mysterious information presented by adults which emphasized procedural skills over all," the adult attempting to negotiate this "return to meaningful learning," and the metacognitive monitor that reflects lucidly on the "mental paralysis" that she experiences even when "one has worked hard and made great effort to control one's reasons and thoughts." By examining Elena's emerging view of conceptual knowledge and her reflections on this process, information can be accessed about viable means of facilitating this process. (Author/MKR)

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# Coming to Terms with Conceptual Knowledge: One Teacher's Journal

**Christine L. Ebert**

Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education

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# COMING TO TERMS WITH CONCEPTUAL KNOWLEDGE: ONE TEACHER'S JOURNAL

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This study will describe the emerging mathematical understanding of one student enrolled in an elementary mathematics content course. Through the medium of her journal entries, a portrait of Elena's struggle to overcome her difficulty with mathematics and construct conceptual knowledge has emerged. Though these struggles are not unique, Elena's story is at once both poignant and extraordinarily compelling. She gives voice to the child within struggling to make sense of this "mysterious information presented by adults which emphasized procedural skills over all," the adult attempting to negotiate this "return to meaningful learning," and the metacognitive monitor that reflects lucidly on the "mental paralysis" that she experiences even when "one has worked hard and made great effort to control one's reasons and thoughts." By examining Elena's emerging view of conceptual knowledge and her reflections on this process, we may also access information about viable means of facilitating this process.

## Theoretical Framework

The current reform movement in mathematics education suggests that teacher subject-matter knowledge is an important component of the new view of mathematical competence. The task of investigating the construction of conceptual knowledge and the subsequent transformation of that subject-matter knowledge into pedagogical content knowledge is extremely complex (Shulman, 1986). As researchers, we struggle to construct tasks that will provide useful information about both of these processes. Within the content and methods classes we seek to design experiences that will facilitate the construction of conceptual knowledge. If we are to provide these opportunities for prospective teachers to construct the conceptual knowledge suggested by the reform documents, then we must examine cases of that construction process through in-depth portraits of individual teachers (Ball, 1988). Elena's journal entries provide important information about her struggles with the mathematics, her reflections on that struggle, and reflections on her mathematical journey.

## Methodology

The data source for this study consists of one prospective elementary teacher enrolled in the first of two courses in mathematical content at a major university in the mid-Atlantic states. The course, which was designed to focus on problem solving and conceptual understanding, consisted of two weekly lectures and two weekly problem solving sessions. This class of approximately 180 students met with the instructor for lectures in a large auditorium and with one of the two teaching assistants for problem-solving sessions in a class of approximately 25 students. The course focused on the "construction of conceptual understanding of the elementary mathematics curriculum" (M251 Course Syllabus, 1993).

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The emphasis on problem-solving was reflected through the allocation of two classes each week to solving problems with a partner. On the first day of problem-solving, all students took a skills test which consisted primarily of 36 arithmetic and pre-algebra problems. Based on the scores from this test, students were assigned a partner for the semester. Pairings were made such that the ability differential for all of the pairs was constant. During the problem-solving sessions students were either assigned problems from the textbook or given a problem-set to work on for that particular day. Problems were never assigned in advance. The role of the instructor during these two weekly sessions was to circulate around the room discussing the problems and problem-solving strategies and offering suggestions whenever appropriate. The "homework" in the course consisted of doing as many problems as necessary to gain understanding of the concepts.

## Results

Elena is a returning adult student in her mid-forties who has worked as an artist and medical illustrator prior to seeking certification to teach art in the elementary school. Elena's journal was chosen for analysis because she consistently recorded her mathematical growth and struggles. She also included a wide variety of other sources from Polya's "little book", *How to Solve It*, to a *New York Times* article about Bob Moses' work on the Algebra Project. All of her quizzes and tests contained additional notes concerning the problems with which she had difficulty and the progress she was able to achieve. The richness of her journal mathematically and the compelling way in which she expresses her reflections throughout this journey provide important and intriguing information about the construction of conceptual knowledge.

### Initiating the Conversation

**Disposition/ Beliefs about Mathematics**. Elena initiated the conversation through the medium of her journal with a quote from Polya's *How to Solve It* that she uses to illustrate her openness to this experience and express the hope that it's "never too late to experience the grain of discovery in the solution of any problem' whenever that susceptible age should occur" (Elena 2/10/93). She also provides her own mathematics history in the initial journal entry.

> It seems amazing that meaningful learning is considered to be "new"! Perhaps it is a new emphasis for mathematics? I certainly was not taught in a way that made mathematics meaningful for me. As a young child I believe that I experienced much of what might be considered the very worst. Math made no sense. The adults presenting this mysterious information emphasized procedural skills over all. "Do it my way, do it correctly and you will obtain the right answer." I am still struggling with the mental/emotional baggage from my earliest introduction to math in elementary school. These words come to

> mind - fear, confusion, discouragement, embarrassment, boredom. Needless to say I do not want to convey any negative attitudes or "hang-ups" to youngsters. I've worked hard as a parent to avoid this and I am striving now to overcome the attitudes that thwart my own progress as a student. (Elena 2/10/93)

**Fears About Doing Mathematics**. While this particular mathematics history is not especially unique, Elena does seem to be able to convey an important distinction between meaningful learning and her own experiences - that the emphasis focused on procedures and that her particular experience generated significant negative feelings. At this point in her reflections it is clear that she is "striving" to overcome the attitudes that thwart her progress. In the following quotation, Elena reflects on the "skills test" and gives voice to the child to describe her initial "return to meaningful learning."

> I took the skills test today. Some things worked, some things didn't. How long has it been since I thought about what an equation is, let alone construct one!...it continues to frustrate me that in math my mind is not at its best - even when I know what to do. It is embarrassing. I often feel that in this realm, I am a nervous 8-yr. old. (Elena, 2/11/93)!

She also writes that she "does not know how to go about describing sequences." A few days later she writes about her attempt to solve the initial problem in the text.

> DESCRIPTION = FORMULA! FORMULA = ALGEBRAIC DESCRIPTION (EXPRESSION) OF SEQUENCE INFORMATION
>
> I find moving methodically, slowly, through these sequences, watching how the numbers "make sense" to be extremely satisfying. I can't remember ever being taught this way. It makes my brain feel good. (Elena 2/15/93)
>
> $n(n+1) = 98,282$
> $n^2 + 1 = 98,282-1$
> $n^2 = 98,281$
> $n = 313$ He is on page 313.

**The Metacognitive Monitor Speaks.** An important contribution of Elena's journal is the multiplicity of perspectives that she conveys as she reflects on these experiences and her attempts to gain conceptual understanding of the mathematics curriculum. Her initial responses are those of the adult negotiating this return to learning and gathering as many and as varied a collection of resources as she is able to make sense of this experience. However, given that her childhood experiences with learning mathematics were so negative, the voice of the frightened, frustrated, and discouraged child is also quite prominent. In addition, when she

can stand back and reflect on the mathematics she is doing, quite frequently the voice is one of metacognitive monitor - dispassionate and reasoned. She writes about examining her thought processes and her response to the first quiz.

> Quiz returned - I did better than expected - what a relief! Making tables is now seared in my brain (as is the formula for the area of a circle - square that radius!). *Determined* to be calmer the next time. (Elena 2/24/93)

On this first "written assessment" Elena did quite well (19/25 or 765%). Although the work and the progress that she makes does not reveal constant growth but the typical fragility inherent in the learning process, she expresses her arrival at this small plateau in an extremely poignant and revealing way.

> Taking in small amounts of information, experiencing small amounts of understanding at an ordered gentle pace is what I need at this time. I almost feel as if something once broken or walled-off is healing or opening. I am amazed by my need for plenty of time and how luxurious it is to be able to return again and again to some small idea and really hold it in my mind...there it is...and it makes sense to me now. (Elena 2/25/93)

This statement is particularly revealing because it stands in such sharp contrast to her previous statements. Although she may have understood intellectually that mathematical ideas could be held in one's mind, considered, and understood, she had not personally experienced this sense of conceptual understanding. Following the return of the quiz, mathematics was still rules and formulas that must be "seared in my brain...square that radius!" rather than concepts and ideas that could be considered and understood. Similarly when she reflects on the value of communicating about mathematics for children, her statement suggests her own wistfulness for this experience. Only in this most recent statement does she indicate a personal and individual understanding that mathematics can indeed make sense to her and the wonderment and sense of peace that she now feels as she experiences learning in this way.

## Observations And Conclusions

It is clear throughout the journal that Elena believes that mathematics should make sense and that doing mathematics should be largely a sense-making proposition. She also indicates a clear understanding of the importance of sufficient time and social interaction in the construction of mathematical knowledge. While this process of struggling with conceptual understanding of the mathematics evokes childhood memories of those same struggles, her ability to reflect on the mathematics from the child's point of view is a real strength. The very nature of these struggles has provided her with the opportunity (she has the natural disposition) to consider central issues in learning and teaching mathematics - the intrinsic value and beauty of mathematics - the importance of constructing knowledge for oneself

- the value of social interaction in the learning process - the importance of children working out their own representations. Elena's journal is also clearly an example of an individual who was pre-disposed to significant self-reflection. It is also unique and idiosyncratic in that she was an "older' student, married, and a mother of two children. She herself makes some of the distinctions between herself and the "twenty-year-olds."

The task of investigating the construction of conceptual knowledge and the subsequent transformation of that subject-matter knowledge into pedagogical content knowledge is extremely complex. As researchers, we struggle to construct tasks that will provide useful information about both of these processes. Elena's journal provides a rich source of just this type of information. Elena's story suggests that serious reflection about learning is very hard and sometimes very painful. However, the value of giving voice to the child within and the metacognitive monitor provides an invaluable lens through which teachers may focus on learning.

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